CONTROL ENGINEERING

Lecture 2

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Fluid Systems

Hydraulic Resistance

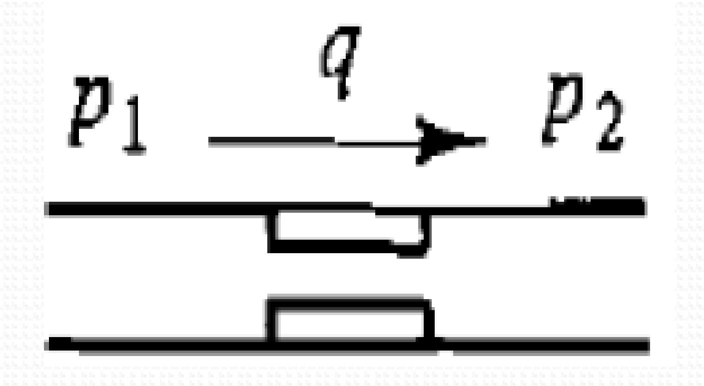
- It is the resistance to flow which occurs as a result of a liquid flowing through valves or changes in a pipe diameter as shown in Figure.
- The relationship between the change in flow rate of liquid *q* through the resistance element and the resulting pressure difference $(p1 - p2)$ is given as

$$R = \frac{\text{Pressure difference}}{\text{Change in flow rate, m}^3/\text{s}} = \frac{p_1 - p_2}{q}$$

Fluid Systems

Hydraulic Resistance

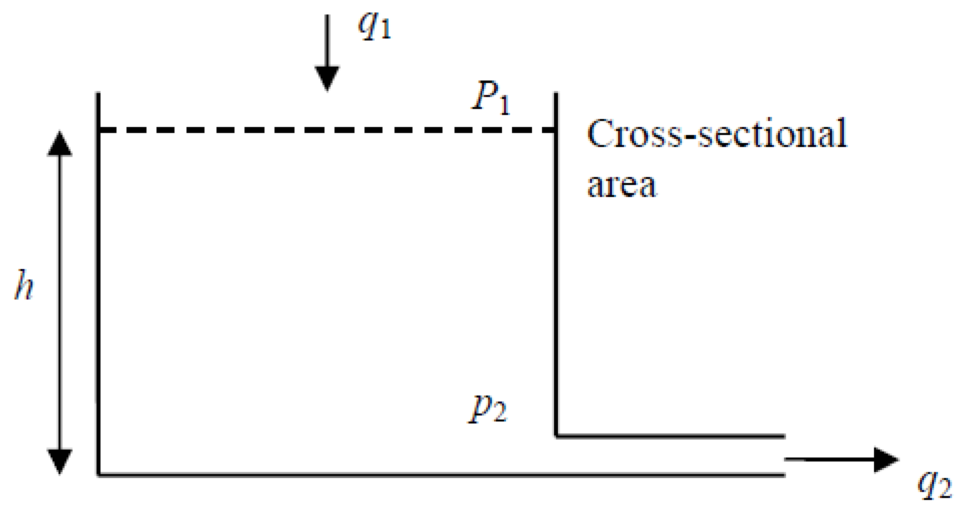
- Pressure drop $p_1 - p_2$ is equivalent to voltage drop and flow rate q to current, R is equivalent to electrical resistance.



Fluid Systems

Hydraulic Capacitance

- It is the term used to describe energy storage with a liquid where it is stored in the form of potential energy as shown in Figure. A height of liquid in the container shown (called pressure head) is one form of such storage.



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- The rate of change of volume V in the tank (dV/dt) is equal to the difference between the volumetric rate at which liquid enters the container $q1$ and the rate at which it leaves the container $q2$

$$q_1 - q_2 = \frac{dV}{dt}$$

- But $V = Ah$, where A is the cross-sectional area of the container and h is the height of liquid in it. Hence

$$q_1 - q_2 = \frac{d(Ah)}{dt} = A\frac{dh}{dt}$$

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- The pressure difference between the input and output is p, where $p = h\rho g$ with ρ being the liquid density and g the acceleration due to gravity.

$$q_1 - q_2 = A\frac{d(p/\rho g)}{dt} = \frac{A}{\rho g}\frac{dp}{dt}$$

- The hydraulic capacitance C is defined as

$$C = \frac{A}{\rho g}$$

Fluid Systems

- Therefore,

$$q_1 - q_2 = C\frac{dp}{dt}$$

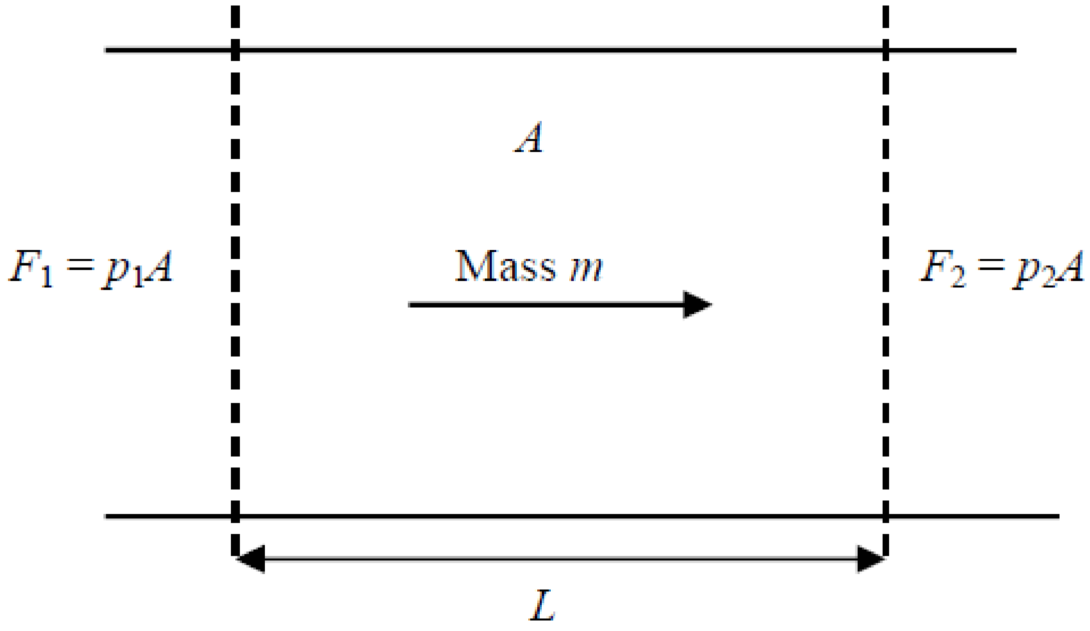
- Integration of above equation gives

$$p = \frac{1}{C}\int (q_1 - q_2)\,dt$$

Fluid Systems

Hydraulic Inertance

- It is the equivalent of inductance in electrical systems or a spring in mechanical systems. To accelerate a fluid and to increase its velocity a force is required.



Fluid Systems

- Consider a block of liquid of mass *m* as shown in Figure. The net force acting on the liquid is

$$F_1 - F_2 = p_1 A - p_2 A = (p_1 - p_2) A$$

- Where ($p1 - p2$) is the pressure difference and *A* the cross-sectional area. This net force causes the mass to accelerate with an acceleration *a*, and so

$$(p_1 - p_2) = ma$$

- However, *a* is the rate of change of velocity *dv/dt*, therefore

$$(p_1 - p_2) A = m \frac{dv}{dt}$$

Fluid Systems

- The mass of liquid concerned has a volume of *AL*, where *L* is the length of the block or the distance between the points in the liquid where the pressures *p*1 and *p*2 are measured. If the liquid has a density ρ then $m = AL\rho$, and

$$(p_1 - p_2)A = AL\rho \frac{dv}{dt}$$

- But the volume rate of flow $q = Av$, hence

$$(p_1 - p_2)A = L\rho \frac{dq}{dt}$$

$$I = \frac{L\rho}{A}$$

$$p_1 - p_2 = I \frac{dq}{dt}$$

Fluid Systems

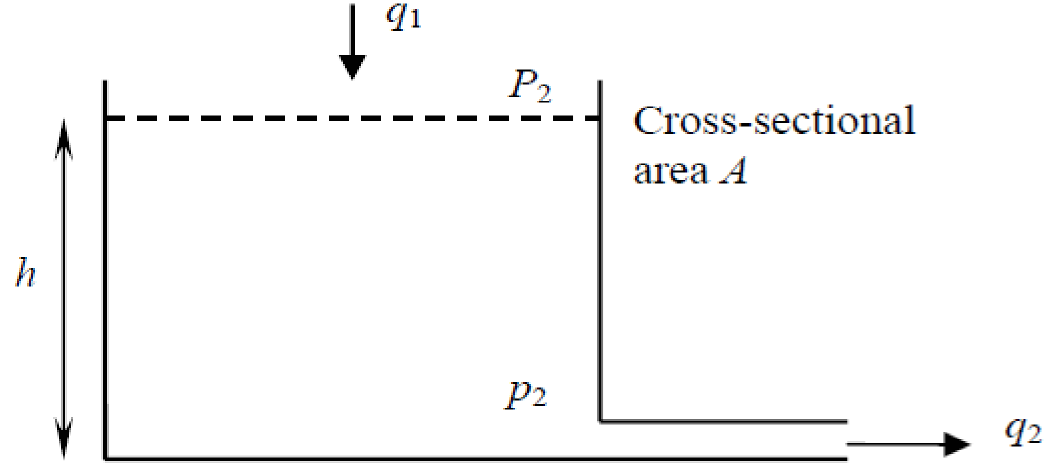
Table 1 Hydraulic building blocks

Building block	Describing equation
Resistance	$q = \frac{p_1 - p_2}{R}$
Capacitance	$q = C\frac{d(p_1 - p_2)}{dt}$
Inertance	$q = \frac{1}{I}\int (p_1 - p_2)dt$

Modeling Hydraulic System

<u>Example 1</u>

- Figure shows a simple hydraulic system as the liquid enters and leaves a container. Such system can be considered to consist of a capacitor, the liquid in the container, with a resistor, the valve. Inertance may be neglected since flow rates change only very slow.



Modeling Hydraulic System

- For the capacitor we can write

$$q_1 - q_2 = C\frac{dp}{dt}$$

- Integration of Equation gives

$$p = \frac{1}{C}\int (q_1 - q_2)dt$$

Modeling Hydraulic System

- The rate at which liquid leaves the container $q2$ equals the rate at which it leaves the valve. Therefore for the resistance R we have

$$R = \frac{p_1 - p_2}{q_2}$$

- The pressure difference ($p1 - p2$) is the pressure due to the height of liquid in the container and is thus $h\rho g$ with ρ being the liquid density and g the acceleration due to gravity. Therefore

$$q_2 = \frac{h\rho g}{R}$$

Modeling Hydraulic System

- Similarly, we may define the capacitance, *C*, of a tank to be the change in quantity of stored liquid necessary to cause a unit change in potential, and so substituting for *q*2 in previous Equation gives

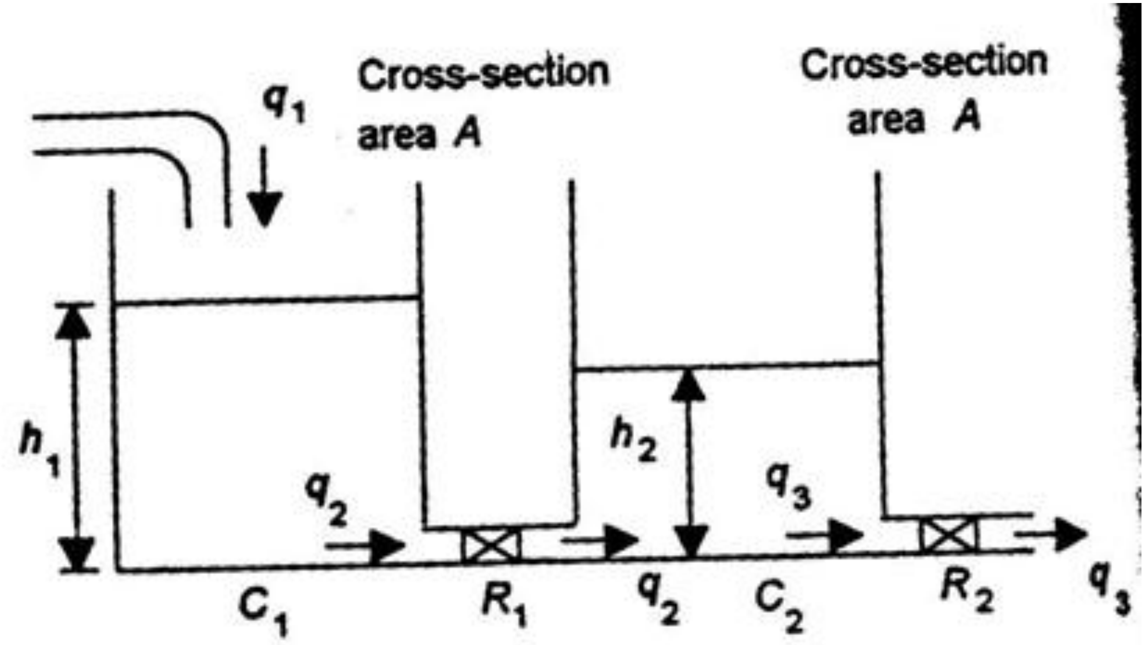
$$q_1 - \frac{h\rho g}{R} = C\frac{d(h\rho g)}{dt}$$

- And, since $C = A/\rho g$

$$q_1 = A\frac{dh}{dt} + \frac{\rho g h}{R}$$

Modeling Hydraulic System

- <u>Example 2</u>



Modeling Hydraulic System

$$p_1 - p_2 = R_1 q_2$$

The pressures are $h_1 \rho g$ and $h_2 \rho g$. Thus

$$(h_1 - h_2)\rho g = R_1 q_2$$

$$q_1 - \frac{(h_1 - h_2)\rho g}{R_1} = A_1 \frac{\mathrm{d}h_1}{\mathrm{d}t}$$

Modeling Hydraulic System

$$q_2 - q_3 = C_2 \frac{dp}{dt}$$

$$q_2 - q_3 = A_2 \frac{dh_2}{dt}$$

$$p_2 - 0 = R_2 q_3$$

$$q_2 - \frac{h_2 \rho g}{R_2} = A_2 \frac{dh_2}{dt}$$

$$\frac{(h_1 - h_2)\rho g}{R_1} - \frac{h_2 \rho g}{R_2} = A_2 \frac{dh_2}{dt}$$

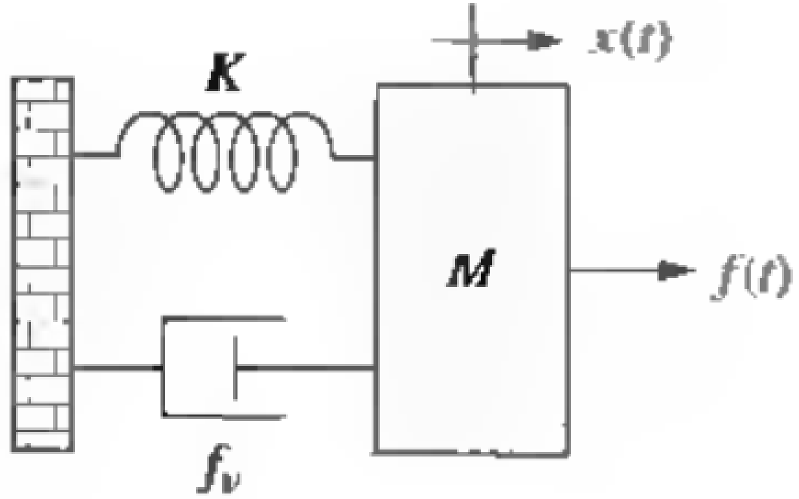
Electrical Circuit Analogs

- The commonality of systems from the various disciplines by demonstrating that the mechanical systems with which we worked can be represented by equivalent electric circuit.
- An electrical circuit that is analogous to a system from another discipline is called an electrical circuit analog.
- Analogs can be obtained by comparing the describing equations, such as the equations of motion of a mechanical system, with either electrical mesh or nodal equations.

Electrical Circuit Analogs

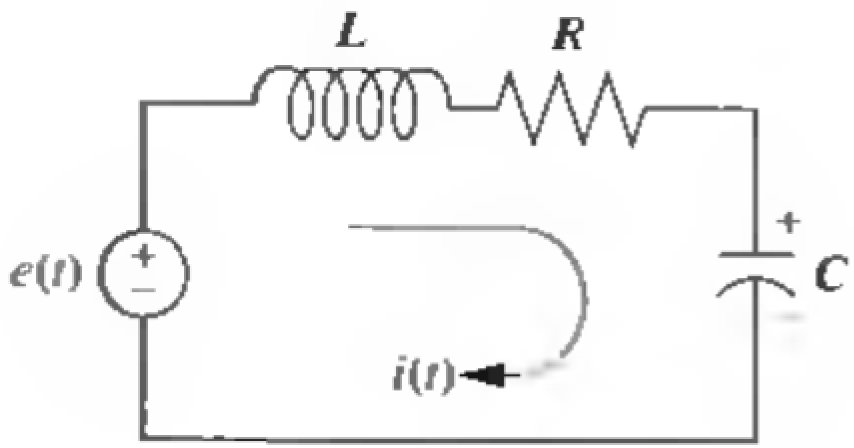
- When compared with mesh equations, the resulting electrical circuit is called a series analog.
- When compared with nodal equations, the resulting electrical circuit is called a parallel analog.

Series Analog



$$(Ms^2 + f_v s + K)X(s) = F(s)$$

Series Analog



$$\left(Ls + R + \frac{1}{Cs}\right)I(s) = E(s)$$

Series Analog

$$(Ms^2 + f_v s + K)X(s) = F(s)$$

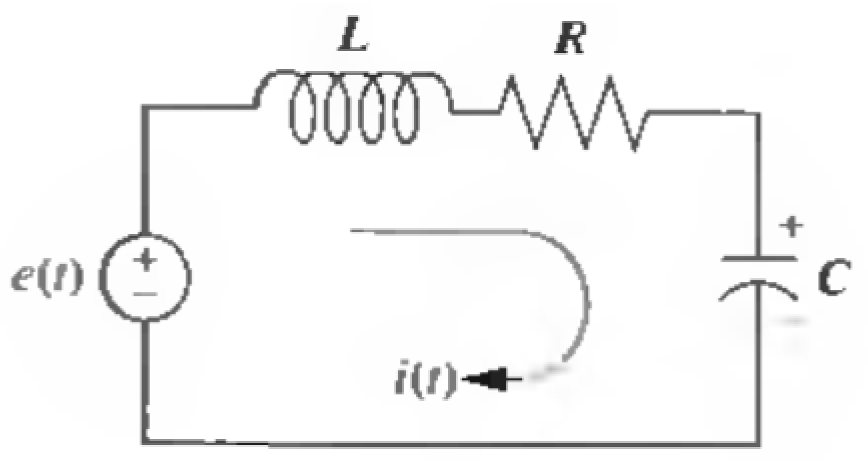
$$\left(Ls + R + \frac{1}{Cs}\right)I(s) = E(s)$$

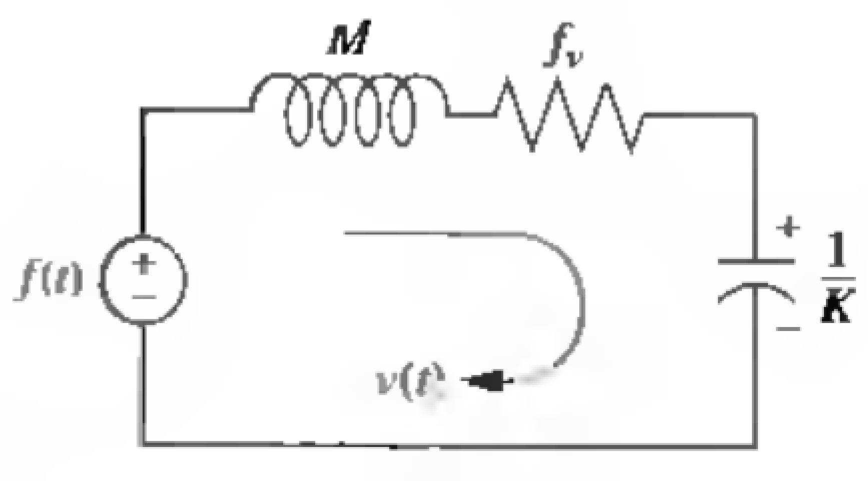
$$\frac{Ms^2 + f_v s + K}{s} sX(s) = \left(Ms + f_v + \frac{K}{s}\right)V(s) = F(s)$$

Series Analog

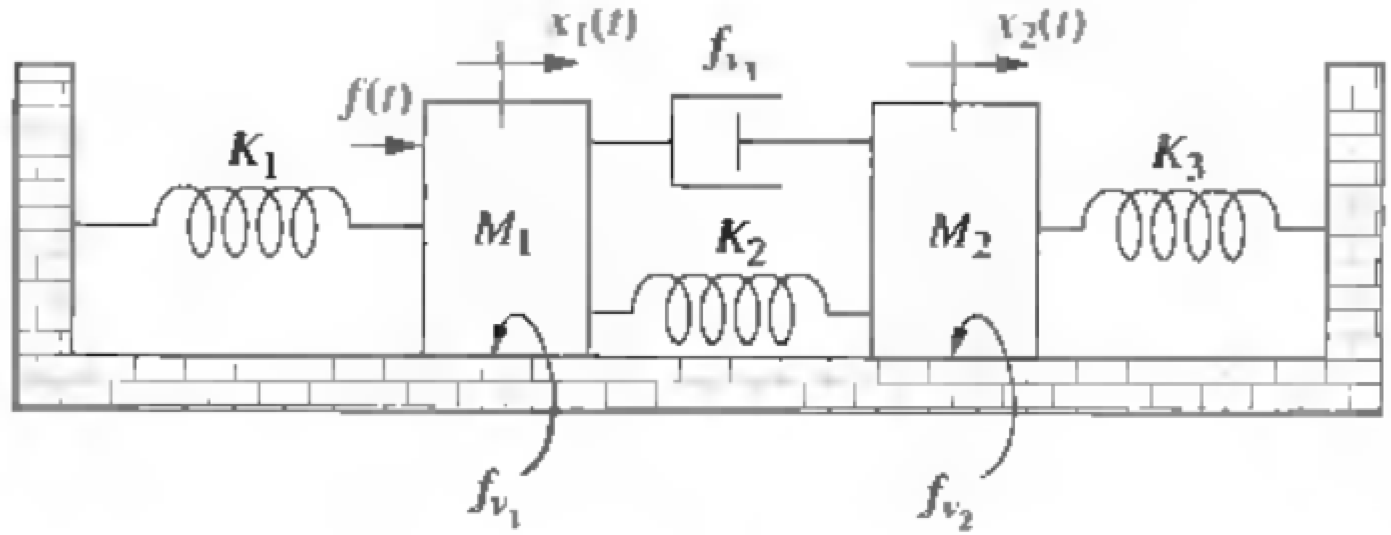
mass =	M	$\longrightarrow$	inductor	$= M$ henries
viscous damper =	f_v	$\longrightarrow$	resistor	$= f_v$ ohms
spring =	K	$\longrightarrow$	capacitor	$= \frac{1}{K}$ farads
applied force =	$f(t)$	$\longrightarrow$	voltage source	$= f(t)$
velocity =	$v(t)$	$\longrightarrow$	mesh current	$= v(t)$

Series Analog





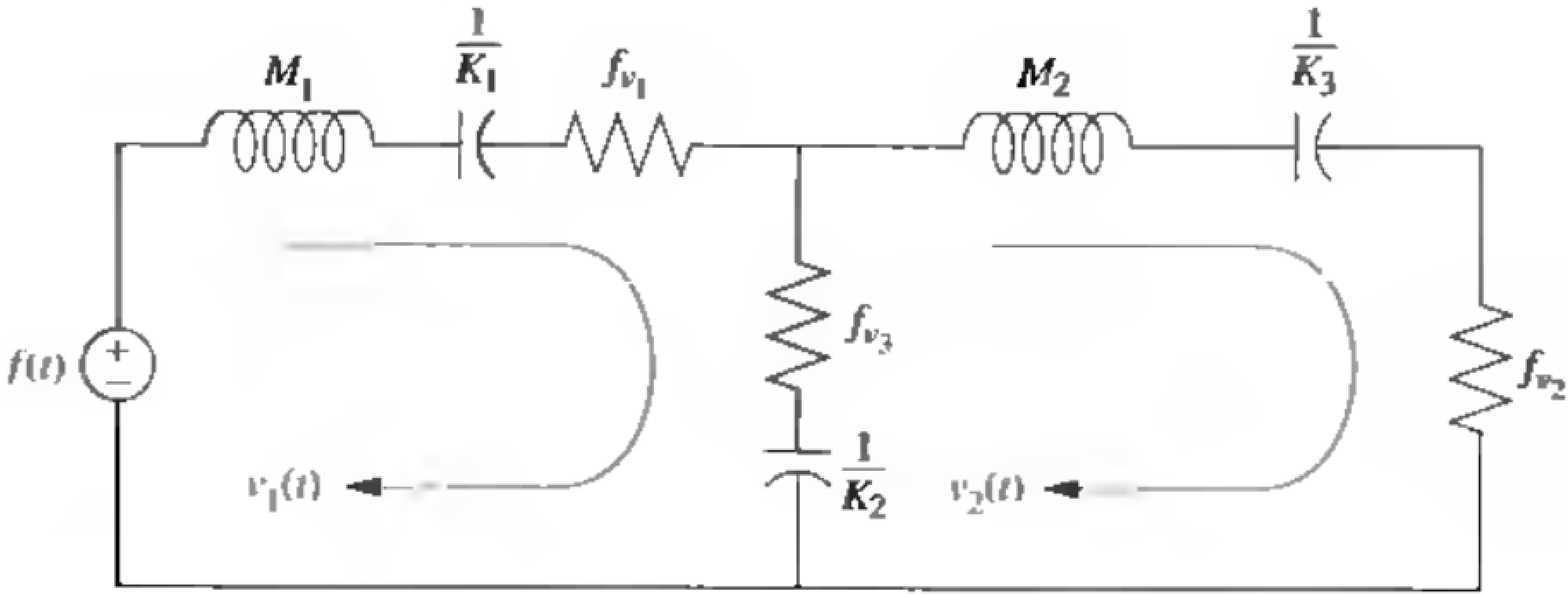
Series Analog



$$\left[M_1 s + (f_{v_1} + f_{v_3}) + \frac{(K_1 + K_2)}{s}\right]V_1(s) \quad - \left(f_{v_3} + \frac{K_2}{s}\right)V_2(s) = F(s)$$

$$-\left(f_{v_3} + \frac{K_2}{s}\right)V_1(s) + \left[M_2 s + (f_{v_2} + f_{v_3}) + \frac{(K_2 + K_3)}{s}\right]V_2(s) = 0$$

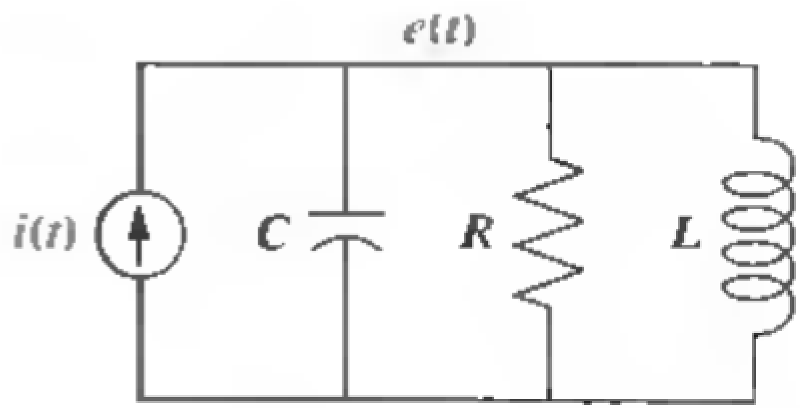
Series Analog



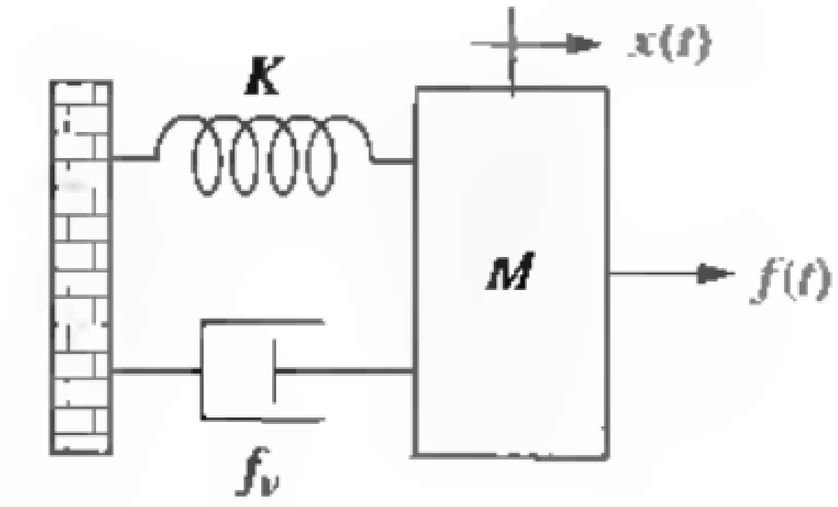
Parallel Analog

- Kirchoff's nodal equation for the simple parallel RLC network shown in figure is

$$\left(Cs + \frac{1}{R} + \frac{1}{Ls}\right)E(s) = I(s)$$



Parallel Analog



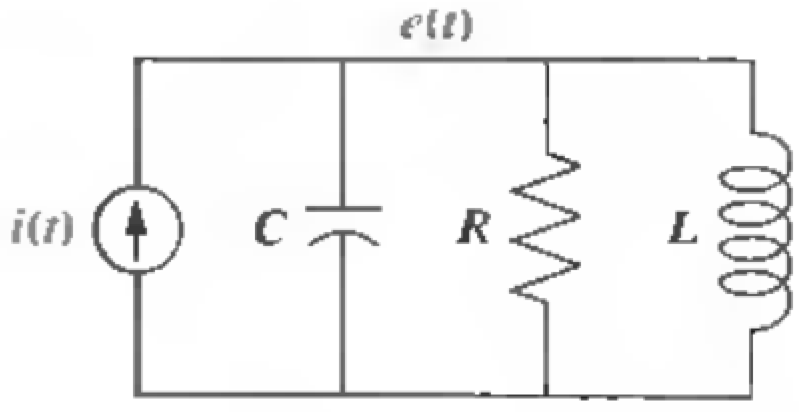
$$\left(Cs + \frac{1}{R} + \frac{1}{Ls}\right)E(s) = I(s)$$

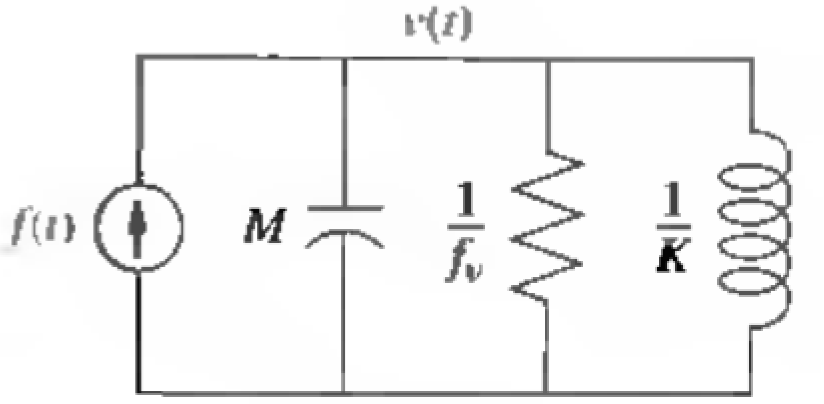
$$\left(Ms + f_v + \frac{K}{s}\right)V(s) = F(s)$$

Parallel Analog

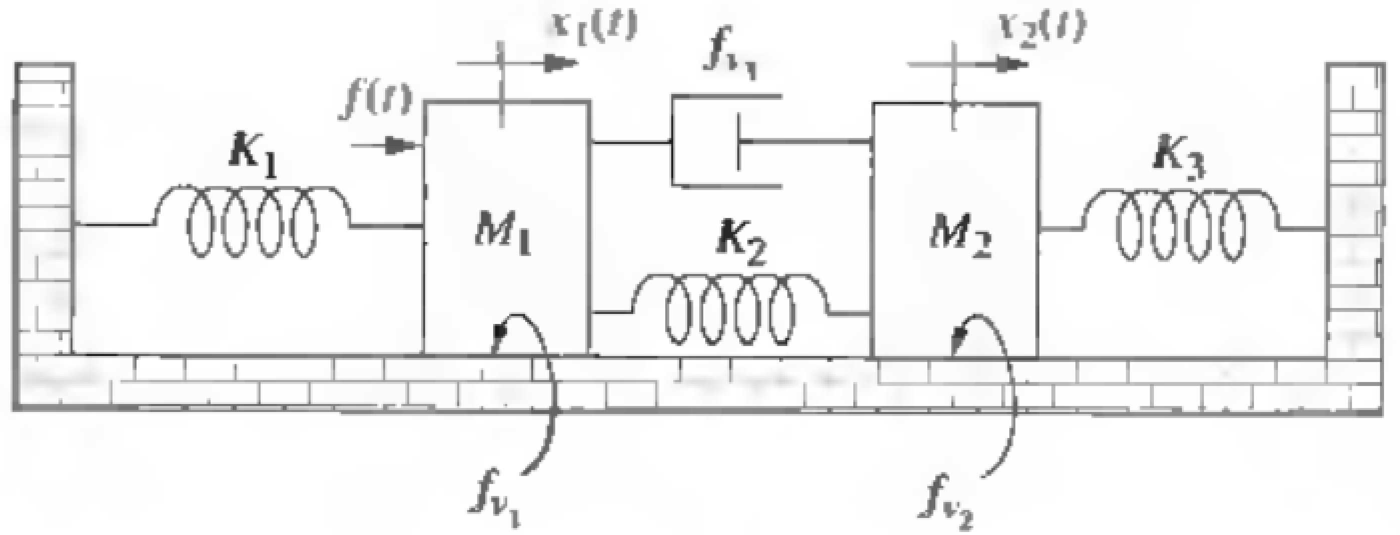
mass =	M	$\longrightarrow$	capacitor	$= M$ farads
viscous damper =	f_v	$\longrightarrow$	resistor	$= \frac{1}{f_v}$ ohms
spring =	K	$\longrightarrow$	inductor	$= \frac{1}{K}$ henries
applied force =	$f(t)$	$\longrightarrow$	current source	$= f(t)$
velocity =	$v(t)$	$\longrightarrow$	node voltage	$= v(t)$

Parallel Analog





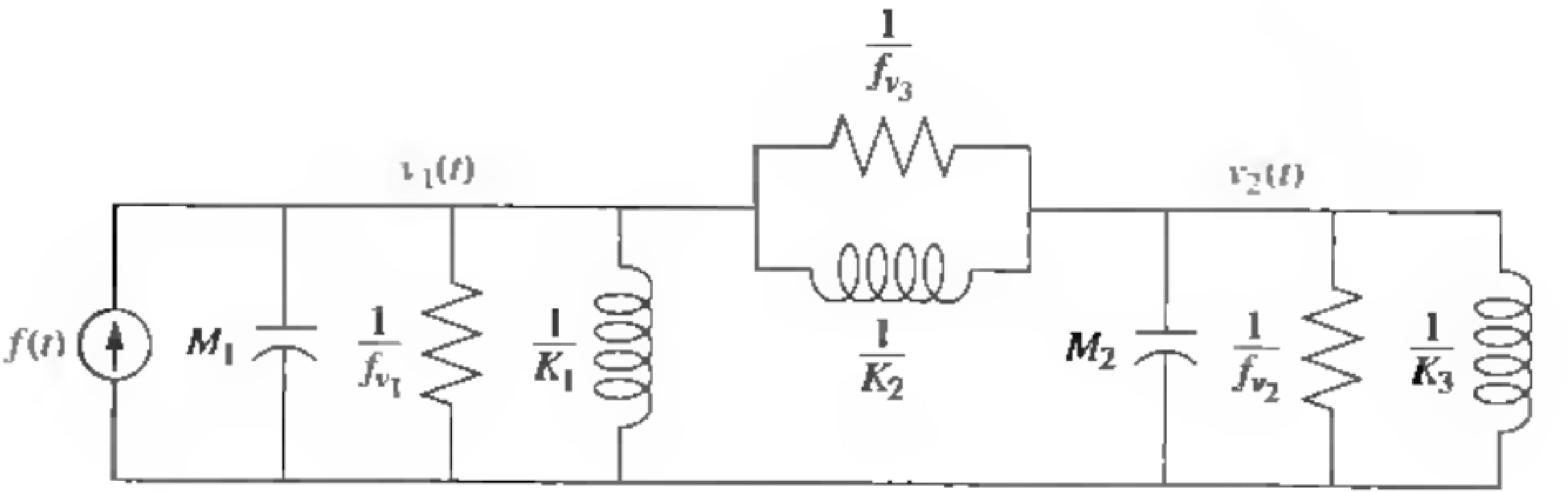
Parallel Analog



$$\left[M_1 s + (f_{v_1} + f_{v_3}) + \frac{(K_1 + K_2)}{s}\right] V_1(s) \quad - \left(f_{v_3} + \frac{K_2}{s}\right) V_2(s) = F(s)$$

$$-\left(f_{v_3} + \frac{K_2}{s}\right) V_1(s) + \left[M_2 s + (f_{v_2} + f_{v_3}) + \frac{(K_2 + K_3)}{s}\right] V_2(s) = 0$$

Parallel Analog



Parallel Analog